Enrollment No: _____Exam Seat No:_____

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Mathematical Physics and Classical Mechanics
Subject Code :4SC05MPC1 Branch: B.Sc. (Physics)
Semester : 5 Date :14/03/2019 Time : 10:30 To 01:30 Marks : 70

Instructions:
(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1		**Attempt the following questions:**	**(14)**
	a)	Define Beta(β) function and give its basic mathematical formulae.	01
	b)	Define Gamma (γ)Give its and its basic mathematical formulae.	01
	c)	Define: Fourier series. Write its general formula and Fourier coefficients.	01
	d)	What are the Dirichlet conditions for a periodic function in Fourier series expansion?	01
	e)	For which type of Fourier series, the coefficients $a_0 \& a_n \neq 0$ and $b_n = 0$?	01
	f)	Draw graph for the Fourier series $f(x) = \begin{cases} -2; 0 < x < 1 \\ 2; 10 < x < 2 \end{cases}$.	01
	g)	Write the value / formula of a_0 , a_n and b_n for the extended intervals.	01
	h)	Write the formula of centripetal acceleration for a bead sliding along the uniformly rotating wire in a free space. Identify the associated physical properties.	01
	i)	Define : Generalised coordinates.	01
	j)	What is virtual displacement?	01
	k)	What do you understand by "Degree of Freedom"?	01
	l)	Define variational principle.	01
	m)	State :D'Alembert's Principle.	01
	n)	Define : Phase Space	01

Attempt any four questions from Q-2 to Q-8

Q-2		**Attempt all questions**	**(14)**
	(A)	Prove the following for the β function, where $p > 0$ and $q > 0$; (1) $\beta(p,q) = 2\int_0^{\pi/2} sin^{2p-1}\theta . cos^{2q-1}\theta . d\theta .$ (2) $\beta(p,q) = \int_0^1 \frac{x^{p-1}+x^{q-1}}{(1+x)^{p+q}}\, dx.$	**08**
	(B)	Express $f(x) = x$ as a half range sine and cosine series in $0 < x < 2$.	**06**
Q-3		**Attempt all questions**	**(14)**
	(A)	Prove the following results for the γ function.	**08**

Where, $p > 0$, $a =$ Constant; $x = y$

(1) $\gamma(p) = \frac{1}{p}\int_0^{\infty} e^{-x^{1/p}}\, dx.$

(2) $\gamma(p) = \int_0^1 \left[\log\left(\frac{1}{y}\right)\right]^{p-1} dy$

(B) Obtain the Fourier series to represent $f(x) = |x|$; where $-\pi < x < \pi$. **06**

Q-4 **Attempt all questions** **(14)**

(A) Find a series of sine and cosine which represents $(x + x^2)$ in the interval $-\pi, x < \pi$ and deduce **07**

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$$

(B) Find the series of sine and cosine functions for **07**

$f(x) = \begin{cases} 0 \ when \ -\pi < x < 0 \\ \left(\frac{\pi}{4}\right)x \ when \ 0 < x < \pi \end{cases}$

and deduce $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

Q-5 **Attempt all questions.** **(14)**

(A) Define: Constraints. Name different types of constraints. Narrate Constrained Forces and their Properties. **07**

(B) Derive equation for the Simple Pendulum from the Lagrangian Formulation. **07**

Q-6 **Attempt all questions** **(14)**

(A) Explain and prove Lagrange's equation of Motion for the nonconservative System. **10**

(B) Derive equation for the Simple Harmonic Motion from the Lagrangian formulation. **04**

Q-7 **Attempt all questions** **(14)**

(A) Derive Hamilton's Equation of motion **07**

(B) Discuss and derive necessary formula for the L-C-R Series and Parallel circuit using Lagrangian formulation and Rayleigh's dissipation function. **07**

Q-8 **Attempt all questions** **(14)**

(A) Obtain Hamilton of simple pendulum with moving support. Also derive formula for the simple pendulum from Hamilton's equation. **07**

(B) Describe an application of Lagrange's undetermined multiplier in quantum mechanical problem of the particle in a box. **07**